Distinguishable Boxes

Examples

1. Suppose I am catering from Yali's and want to buy sandwiches to feed 60 students. How many ways can I do this if they have 8 sandwich options? How many ways can I do this if I want to get at least 2 of each sandwich?

2. How many ways can I distribute 30 course spots amongst 4 grades (freshmen, sophomores, juniors, seniors) so that there are no more than 10 freshmen in the course?

Problems

3. True   False   When we are counting the number of bitstrings of 0 and 1 with a certain number of 0's, the ordering of the 0 and 1's matter which means that we should use $P()$ as opposed to $C()$.

4. True   False   In Example 1, since each student is getting a sandwich, the balls are sandwiches and the urns are students.

5. In the card game Sheng Ji or 80 points, two decks of a total of 108 cards are dealt out to 4 people such that each person gets 25 cards and there are 8 cards left over. How many ways can this occur?

6. How many solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ if all are positive integers and $x_3 \leq 3$?

7. How many 7 digit decreasing numbers are there? One example is 9777650.

8. How many 3 digit numbers have a sum of digits equal to 9?

9. How many numbers less than $1,000,000$ have the sum of their digits equal to 10?

10. How many ways can you deal the 52 cards of a deck to 4 people so that everyone gets 13 cards and the oldest player gets the queen of spades?

Indistinguishable Boxes

Example

11. How many ways are there to split 28 distinct students up into at most 6 different groups if the groups are not numbered? What if the students are not distinct?

Problems

12. True   False   The only way to determine what $S(5,3)$ is to list out all the possibilities.

13. True   False   The only way to determine what $p_3(5)$ is to list out all the possibilities.

14. True   False   In order to determine the number of ways to distribute 10 distinguishable items into 3 identical boxes so that each box has at least 2 items, we can place one item in each box and this problem reduces to the regular case of distributing 7 items in 3 identical boxes which is $S(7,3)$ ways.

15. True   False   In order to determine the number of ways to distribute 10 identical items into 3 identical boxes so that each box has at least 2 items, we can place one item in each box and this problem reduces to the regular case of distributing 7 items in 3 identical boxes which is $p_3(7)$ ways.

16. There are 14 students that want to break off into 3 non-empty study groups. How many ways can this occur?

17. I want to store my 200 Yu-gi-oh cards in 4 different identical boxes. How many ways can I do this if some boxes are allowed to be empty?

18. How many ways are there to split 15 identical marbles to 5 different non-empty groups?

19. How many ways are there of distributing 30 identical objects into 3 boxes if each box must have at least 5 items?